DOI: https://doi.org/10.5281/zenodo.14556612

PARABOLIK TIPDAGI TENGLAMA UCHUN KOSHI MASALASI

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Annotatsiya: *Ushbu maqolada parabolik tipdagi tenglama uchun Koshi masalasi va uning xususiy yechimlari toppish qisqacha bayon etilgan.*

Kalit so‘zlar: *Koshi masalasi, xususiy yechimlar, issiqlik tenglamasi.*

Bizga cheksiz to‘g‘ri chiziqda bir jinsli issiqlik tarqalish tenglamasi

$$u_t = u_{xx}, -\infty < x < \infty, t > 0 \qquad (1)$$

berilgan bo‘lsin. (1) - tenglama uchun

$$u(x,t)|_{t=0} = x \cdot e^{-x^2} \qquad (2)$$

$-\infty < x < \infty$ Koshi shartini qanoatlantiruvchi yechimi topilsin.

Dastlab, yordamchi

$$u_t = u_{xx} \quad (1),$$

$$u(x,t)|_{t=0} = \varphi(x), \quad -\infty < x < \infty \quad (3)$$

cheksiz to‘g‘ri chiziqda koshi masalasini yechamiz. Yechimni $u(x,t) = X(x)T(t)$

ko‘rinishida izlaymiz. Bundan quyidagilarni aniqlaymiz:

$$u_{xx} = X''(x)T(t), \quad u_t = X(x)T'(t), \quad X(x)T'(t) = X''(x)T(t)$$

$$\frac{T'(t)}{T(t)} = \frac{X''(x)}{X(x)} = -\lambda^2, \qquad \begin{cases} X''(x) = -X(x)\lambda^2 \\ T'(t) + \lambda^2 T(t) = 0 \end{cases}$$

$$X''(x) + X(x)\lambda^2 = 0$$

bu tenglamaning yechimi quyidagidan iborat:

$$X_1 = e^{-i\lambda x}, \ X_2 = e^{i\lambda x}.$$

Endi ikkinchi tenglamani yechaylik:

$$\frac{dT}{T} = -\lambda^2 dt \ \Rightarrow \ lnT = -\lambda^2 t + C \ \Rightarrow \ T(t) = e^{-\lambda^2 t} * C$$

$$u(x,t) = \sum_{n=-\infty}^{\infty} u_n(x,t) = \sum_{\lambda=-\infty}^{\infty} T_\lambda(t) X_\lambda(t) = \int_{-\infty}^{+\infty} A(\lambda)\, e^{-\lambda^2 t} \cdot e^{\pm i\lambda x} d\lambda$$

Bunda λ ixtiyoriy xaqiqiy son, shuning uchun so‘nggi integralda $+$ ishoraligini olamiz.

$$u(x,t) = \int_{-\infty}^{+\infty} A(\lambda)\, e^{-\lambda^2 t + i\lambda x} d\lambda$$

Yordamchi masala (3) shartiga ko‘ra $t = o$ da

$$\varphi(x) = \int_{-\infty}^{+\infty} A(\lambda)\, e^{-i\lambda x} d\lambda$$

Furye almashtirishini qo‘llasak,

$$A(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \varphi(\xi)\, e^{-i\lambda\xi} d\xi$$

$$u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} \varphi(\xi)\, e^{-i\lambda\xi} d\xi \right) e^{-\lambda^2 t + i\lambda x} d\lambda =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} e^{-\lambda^2 t + i\lambda(x-\xi)}\, d\lambda \right) \varphi(\xi) d\xi$$

Ichki integralni hisoblaymiz:

$$I = \int_{-\infty}^{+\infty} e^{-\lambda^2 t + 2i\lambda\frac{\sqrt{t}(x-\xi)}{2\sqrt{t}} - \frac{i^2(x-\xi)^2}{(2\sqrt{t})^2} - \frac{(x-\xi)^2}{(2\sqrt{t})^2}} d\lambda =$$

$$= \frac{1}{\sqrt{t}} \int_{-\infty}^{+\infty} e^{-\left(\lambda\sqrt{t} - \frac{i(x-\xi)}{2\sqrt{t}}\right)^2} d\left(\lambda\sqrt{t} - \frac{x-\xi}{2\sqrt{t}}\right) e^{-\left(\frac{x-\xi}{2\sqrt{t}}\right)^2} =$$

$$= \frac{1}{\sqrt{t}} \sqrt{\pi}\; e^{-\left(\frac{x-\xi}{2\sqrt{t}}\right)^2}$$

$$u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\sqrt{\pi}}{\sqrt{t}} e^{-\left(\frac{x-\xi}{2\sqrt{t}}\right)^2} \varphi(\xi) d\xi = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\left(\frac{x-\xi}{2\sqrt{t}}\right)^2} \varphi(\xi) d\xi \qquad (4)$$

Ichki integralni hisoblashda

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi} \qquad (**)$$

tenglikdan foydalandik. Ushbu tenglikni isbotlash uchun $x = \lambda y$ ko‘rinishida ifodalaymiz. $dx = \lambda dy$ ekanligini hisobga olsak,

$$I = \int_{-\infty}^{+\infty} e^{-x^2} dx = 2\int_{0}^{+\infty} e^{-x^2} dx = 2\int_{0}^{+\infty} e^{-\lambda^2 y^2} \lambda dy$$

tenglikni ikki tomonini $e^{-\lambda^2}$ ko‘paytirib, $(0, +\infty)$ integrallaymiz.

$$\frac{I}{2} = \int_{0}^{+\infty} e^{-\lambda^2 y^2} \lambda dy$$

$$\frac{I}{2} \int_{0}^{+\infty} e^{-\lambda^2} dy = \int_{0}^{+\infty} \int_{0}^{+\infty} e^{-\lambda^2 y^2} e^{-\lambda^2} \frac{d\lambda^2}{2} dy$$

$$\frac{I}{2}\frac{I}{2}=\frac{1}{2}\int_{0}^{\infty}\frac{dy}{1+y^2}\int_{0}^{\infty}e^{-\lambda^2(y^2+1)}d{-}\lambda^2(y^2+1)$$

$$\left(\frac{I}{2}\right)^2=-\frac{1}{2}\int_{0}^{\infty}darctgy\left(e^{-\lambda^2(y^2+1)}\Big|_{0}^{\infty}\right)$$

$$\left(\frac{I}{2}\right)^2=\frac{1}{2}\int_{0}^{\infty}darctgy$$

$$\left(\frac{I}{2}\right)^2=\frac{1}{2}(arctg\infty\ -\ arctg0)$$

$$\frac{I^2}{4}=\frac{1}{2}\left(\frac{\pi}{2}-0\right)\ =>\ \frac{I^2}{4}=\frac{\pi}{4}\ \Longrightarrow\ I=\sqrt{\pi}$$

Demak,

$$\int_{-\infty}^{+\infty}e^{-x^2}dx=\sqrt{\pi}$$

(1) tenglama yechimini (4) dan foydalanib, Koshi masalasini yechamiz:

$$u(x,t)=\frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{+\infty}e^{-\left(\frac{x-\xi}{2\sqrt{t}}\right)^2}\xi e^{-\xi^2}d\xi$$

bo‘laklab integrallash natijasida, ushbu yagona yechim kelib chiqadi:

$$u(x,t)=\frac{x}{\sqrt{(4t+1)^3}}e^{-\frac{x^2}{4t+1}}$$